U-Substitution

1. Determine if U-Substitution is appropriate.
   a. Do you see a **function inside of another function**?
   b. Is there a **function being divided by another function**?

   These aren't the only rules of thumb, but these are good indicators
2. Find your $u = g(x)$
   a. Try using the inside function first or if you can't see what to use, start simple (just don't use $u = x$) and work up in complexity
   b. Substitute $u = g(x)$ into the integral and see what is left
      i. Do you think the parts remaining could be found by taking the derivative of $u = g(x)$?
      ii. If you don't think you will get the remaining parts by taking the derivative, increase the complexity of $\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{x})$ and try again
3. Differentiate u with respects to x
   a. You will get something like $\frac{du}{dx} = stuff$
   b. Determine what you want to see on one side of this $\frac{du}{dx} = stuff$
      i. Take the original integral
      ii. Substitute $u = g(x)$ into it
      iii. Take all of the remaining parts with $x's$
      iv. These remaining x parts is what you want to get on one side of the equation from $\frac{du}{dx} = stuff$
      v. After you get all of these parts on one side of the equation you should have no $\boldsymbol{x's}$ on the other side. If you do, you will need to choose anther $\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{x})$
      vi. If you can't get all of the parts to one side and you are stuck having $\boldsymbol{x's}$ on both sides, you will need to choose anther $\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{x})$
4. Substitute your $u = g(x)$ and the parts from $\frac{du}{dx} = stuff$
   a. Do you only have $\boldsymbol{u's}$ in the equation? If yes, integrate, if not you will need to choose another $\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{x})$
5. Integrate the integral that should only have $u's$
6. Substitute $u = g(x)$

Two Options for Definite Integrals

6. Substitute $u = g(x)$	6. Convert a and b by taking $g(a) = u_a$ and $g(b) = u_b$
7. Take $F(a) - F(b)$	7. Without substituting $u = g(x)$ Take $F(u_a) - F(u_b)$

Integration by Parts

1. Determine if integration by parts is appropriate.
    a. Do you see a **function being multiplied by another function**?
    b. Does **u-substitution look like it won't work?**
        i. If you choose a function "inside" another function and take its derivative will there be more x's left in the equation
2. Find the functions $f(x)$ and $g'(x)$ being multiplied together
    a. <u>Make $f(x)$ easy to derive</u>
    b. <u>Make $g'(x)$ easy to integrate</u>
        i. If both functions are easy to integrate choose the one that produces the easiest integral to work with
3. Set, $u = f(x)$ and $\frac{dv}{dx} = g'(x)$
4. Create the integration matrix
    a. Rewrite $\frac{dv}{dx} = g'(x)$ such that we have $dv = g'(x)dx$ (now we can take its integral)

$$\boldsymbol{u = f(x)} \quad \Rightarrow$$

$$\Leftarrow \quad \boldsymbol{dv = g'(x)dx}$$

5. Derive $u = f(x)$ and integrate $dv = g'(x)dx$
    a. Rewrite $\frac{du}{dx} = f'(x)$ as $du = f'(x)dx$

$$\boldsymbol{u = f(x)} \quad \Rightarrow \quad \boldsymbol{du = f'(x)dx}$$

$$\boldsymbol{v = g(x)} \quad \Leftarrow \quad \boldsymbol{dv = g'(x)\,dx}$$

6. Substitute u, du, v, and, dv into our given equation

$$\boldsymbol{\int u\,dv \; = u \cdot v \; - \int v\,du}$$

7. Integrate the easy integral

$$\int v\,du$$

8. Combine all elements into one final equation (no need to simplify) **+C**

$$\boldsymbol{\int f(x) \cdot g'(x)\,dx \; = f(x) \cdot g(x) - \int g(x) \cdot f'(x)\,dx + c}$$

9. Definite integral

$$\boldsymbol{\int_a^b u\,dv \; = u \cdot v|_a^b - \int_a^b v\,du}$$